Problem set 3: Imaging performance metrics

Theoretical image science
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Discussing solutions is encouraged but you must **individually** write and hand in your solutions. For numerical computations and plots, you can use the programming language of your choice, e.g. Matlab or Python.

These homework problems build on material covered in **lectures 1-4** as well as the video lecture on NEQ and DQE.

1 Nonprewhitening observer (2p)

As in the lecture, let $q(\mathbf{r})$, $\mathbf{r} \in \mathbb{R}^2$ be an input signal, $\mathcal{H}$ a linear, shift-invariant system with transfer function T and $n(\mathbf{r})$ a wide-sense stationary random process with zero mean. Let $d(\mathbf{r}) = \mathcal{H}[q](\mathbf{r}) + n(\mathbf{r})$ be the system output (the image).

The nonprewhitening observer has the test statistic $\mathbf{w}^t\mathbf{d}$ where the template $\mathbf{w} = \boldsymbol{\Delta}\overline{\mathbf{d}}$ equals the signal difference.
a) Show that the squared detectability is given by

$$d'^2 = \frac{\overline{d}^2 \left[\int_{\mathbf{k}} |\Delta S(\mathbf{k})|^2 \mathrm{MTF}^2(\mathbf{k}) \mathrm{d}\mathbf{k}\right]^2}{\int_{\mathbf{k}} |\Delta S(\mathbf{k})|^2 \mathrm{MTF}^2(\mathbf{k}) \mathrm{NPS}(\mathbf{k}) \mathrm{d}\mathbf{k}} \tag{1}$$

with $\Delta S(\mathbf{k}) = \Delta Q(\mathbf{k})/\overline{q}$. (NPS refers to the NPS of $d(\mathbf{r})$.)
b) Show that (1) is equal to the corresponding expression for the Hotelling observer $d'^2 = \int_{\mathbf{k}} \mathrm{NEQ}(\mathbf{k}) |\Delta S(\mathbf{k})|^2 \mathrm{d}\mathbf{k}$ if the noise is white.

2 Channelized Hotelling observer (2p)

Let $\mathbf{t}_i, i = 1, \ldots, N$ be a set of channel filters used in a channelized Hotelling observer without internal noise. Let $K_{\mathbf{g}}$ be the covariance matrix of the signal and $s = \Delta\overline{\mathbf{g}}$ the signal difference. Show that the detectability (=SNR) of the channelized Hotelling observer d'_{CHO}

is equal to that of the Hotelling observer without channels d'_{Hot} if $K_{\mathbf{g}}^{-1}\mathbf{s}$ can be expressed as a linear combination of $\mathbf{t}_i$. Hint: use Eg. 11.31 in the book [BKVM vol 1].

3 Detector modeling (3p)

A somewhat simplified model of a planar gamma-ray imaging system is as follows.

1. A gamma photon enters the detector at position $\mathbf{r} = (x, y)^t$ and interacts with probability η. If it does not interact it simply passes through the detector.
2. If the photon interacts, it transfers all of its energy to an electron that travels some distance through the material before generating a cloud of positive and negative charges centered at $\mathbf{r}_{\mathrm{centroid}} = \mathbf{r} + \mathbf{\Delta r}_1$. Here $\mathbf{\Delta r}_1$ is random variable with a rotation-symmetric probability distribution such that $|\mathbf{\Delta r}| \in \exp(a)$. (Exponential distribution - look it up!)
3. The charge cloud is registered and the detector electronics assigns an estimated location to the event $\hat{\mathbf{r}} = \mathbf{r}_{\mathrm{centroid}} + \mathbf{\Delta r}_2$ with $\mathbf{\Delta r}_2 \in \mathscr{N}(\mathbf{0}, b\mathrm{I})$ with $\mathbf{I}$ being the identity matrix.
4. Finally, the image is displayed as a continuous (infinitely zoomable) image by putting a Gaussian "blob" of standard deviation c (same in all directions) at the center of each estimated photon position $\hat{\mathbf{r}}$.

a) Derive expressions for the MTF and the large-area gain factor of this imaging system.
b) derive the NPS in the case when a Poisson-distributed number of input photons with expected value λ are incident on the system. Assume that the photons enter the detector at random, uncorrelated locations.
c) Use the above quantities to derive a formula for the NEQ of the system.
d) Derive a formula for the DQE of the system. Note which of the parameters a, b and c the DQE depends on and explain why.

4 DQE in electron microscopy (3p)

Fig. 1 shows the detective quantum efficiency of an electron detector with 55 µm pixel size (center-to-center). If this detector is used in a transmission electron microscope with a magnification of $1.0 \cdot 10^5$, calculate the fluence (electrons per area) measured at the detector, i.e. after magnification, that must be incident on the detector in order to detect a Gaussian feature with a standard deviation of 0.36 nm if the transmission at the center of the Gaussian is reduced by 10% relative to the surroundings. Assume that human detection performance is well approximated by a Hotelling observer with a detection threshold of $\mathrm{SNR}_{\mathrm{Hot}} = 5$. Also assume that the DQE curves in the x and y directions are identical. Hint: you can use `ginput` in matlab or python (`matplotlib.pyplot` to transform a figure into a vector of numbers with a few mouse clicks.

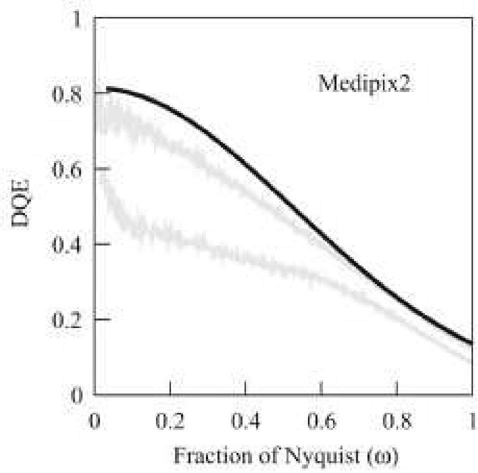

Figure 1 – The black curve is the detective quantum efficiency of an electron detector (Medipix2, with 55 µm pixels). Reproduced from G. McMullan, et al. "Detective quantum efficiency of electron area detectors in electron microscopy", Ultramicroscopy 109 (9), pp. 1126-1143, 2009. (CC BY 3.0)