Problem set 4: Model observers and analytical image reconstruction

Theoretical image science
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Discussing solutions is encouraged but you must **individually** write and hand in your solutions. For numerical computations and plots, you can use the programming language of your choice, e.g. Matlab or Python.

These homework problems build on material covered in **lectures 1-5** as well as the video lecture on NEQ and DQE.

1 Measure your own detection efficiency(5p)

In this computer lab you will measure your own (or someone else's) efficiency for detecting circular disks in white noise. In this assignment, you are expected to submit the code together with a short summary of the results.

First, write a computer program that generates pairs of images of 400×400 pixels containing white noise with mean 0 and standard deviation 1. Then add a circular filled disc with a radius $r = 40$ pixels on the center of one of the images in each pair. The region inside the disc will still contain noise but will have a mean of g which is different from the mean of 0 in the surroundings.

a) Calculate SNR and AUC for the Hotelling observer for the task of detecting this disc (as an SKE/BKE task). Also, how does this tell you the percentage of correct responses in a 2AFC trial for the Hotelling observer?

b)Use the known template (a noiseless image with a disc) to perform a 2AFC trial with the Hotelling observer and report the percent correct (PC). Use $g = 0.01$ and use 500 trials (=pairs of images). You can also use fewer trials if it takes too much time to run. Compare to the number predicted according to a).

c) As you can see if you plot the images, it's very hard to see the insert in these images. Therefore, we'll increase the contrast by changing to $g = 0.05$. This is still hard to see but not impossible for a human. Plot the signal-resent and signal-absent images side by side in random order and find a color scale (`caxis` in Matlab, `pyplot.clim` in python) that makes it as easy as possible to see the feature.

d) Now perform a human reader study, either with yourself or with your friend as test person. The person can be anonymous i.e. you don't need to say who the test person is. Use 50 trials with $g = 0.05$ and find the percent correct. Also calculate the detectability (or "SNR") $d_A = 2\text{erf}^{-1}(2\text{AUC} - 1)$ of the human observer and calculate the efficiency relative to the hotelling observer $\eta = d_A^2/\text{SNR}_{\text{Hot}}^2$. (You have to use the theoretical Hotelling SNR here, we have too little statistics to measure it empirically.)

Note: don't be upset if your efficiency is significantly less than half that of a Hotelling observer. Efficient reading requires optimal viewing conditions and training! Also, we did not include markers to show the location of the feature.

2 Free-response receiver operating characteristic (2p)

In this problem we will study a simple "toy" model of a free-response task. The setting resembles problem HW2.1: Assume that we have a $N \times N$ pixel image of the night sky, where the expected pixel value is $\overline{g}_i = 1$ if there is a star in the pixel and $\overline{g}_i = 0$ otherwise. Assume that there are N_* stars in the image. The image is corrupted by white noise with standard deviation σ. However, this time there may be several stars in the image and the task is to identify all pixels that contain stars.

Assume that this is done with a simple observer that selects all pixels that have a threshold higher than t. Derive formulas for the fraction of correctly localized stars and the mean number of false positives per image. Also plot the FROC curve based on these expressions. You may need to express the quantities on the axes in terms of N and N_*.

3 Tomosynthesis reconstruction (3p)

In some cases, such as in electron microscopy and mammography, it may not be possible to take images from all directions so that a tomographic image has to be reconstructed from a limited angle range. This reconstruction technique is called tomosynthesis. Assume that the image of an object $f(x, y)$ is reconstructed from its radon transform but that only a limited set of projection angles $\theta \in [0, \pi/2]$ are available. Further assume that the other projections are set to zero and that image is reconstructed by filtered backprojection.

Show that the resulting image is $\hat{f}(x, y) = f(x, y) * h(x, y)$ where

$$h(x, y) = \frac{1}{2}\left(\delta(x)\delta(y) - \frac{1}{\pi^2 xy}\right) \tag{1}$$

Hint: use the Fourier slice theorem. You will have to find the inverse Fourier transform of

a function that equals 1 in two quadrants and 0 in the others.
Note: This way of reconstructing limited-angle scans gives severe artifacts in the image so it can be a good idea to modify the reconstruction algorithm, for example by changing the ramp filter.